# Intermittency and turbulence in a magnetically confined fusion plasma

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We investigate the intermittency of magnetic turbulence as measured in reversed field pinch plasmas. We show that the probability distribution functions of magnetic field differences are not scale invariant; that is, the wings of these functions are more important at the smallest scales, a classical signature of intermittency. We show that scaling laws appear also in a region very close to the external wall of the confinement device, and we present evidences that the observed intermittency increases moving towards the wall.

The issue of self-similarity is of paramount importance in turbulence studies. Indeed, self-similarity is one of the key hypotheses of Kolmogorov theory [1,2], which leads for fluid turbulence to the famous $-5/3$ exponent for the power spectrum decay in the inertial range (the intermediate range of scales between the large scales where energy is injected and the small ones where it is dissipated). Notwithstanding the success of the Kolmogorov theory, the study of the probability distribution functions (PDF) of velocity fluctuations at a given scale has shown a departure from Gaussianity in the PDF tails. The same phenomenon is usually evidenced by looking at the scaling exponents of higher order moments of fluctuations, which appear to be nonlinear functions of the order index. Intermittency described by multifractal models [2] is usually invoked to be the cause of the observed break of pure self-similarity. Even if Kolmogorov theory was originally developed for homogeneous and isotropic turbulence, the evidence for self-affine fields has been studied also inside the boundary layer turbulence in fluid laboratory experiments [3], which is neither homogeneous nor isotropic. The only hypothesis required to perform these studies from an experimental point of view, in order to apply the usual Taylor's hypothesis, is the statistical stationarity of turbulence. Recently, it has been shown [4] that only after a suitable decomposition in terms of irreducible representations of the SO(3) groups one can hope to properly disentangle isotropic from anisotropic effects in Navier-Stokes equations. Of course, this should be true also in magnetohydrodynamic (MHD) flows, even if it is not clear how to recover anisotropic and nonhomogeneous effects from real data.

While in ordinary fluids the statistical properties of turbulence have been well characterized, both theoretically and experimentally, in magnetized fluids only recently this has been undertaken, mostly in relation with velocity and magnetic field fluctuations measured in the solar wind [5]. In this Rapid Communication we report evidences for the presence of intermittency in another type of magnetized fluid, namely, a plasma of interest for controlled thermonuclear fusion research, confined in reversed field pinch (RFP) configuration.

The RFP is a configuration of magnetic fields [6] characterized by toroidal and poloidal components of comparable magnitude (in a tokamak the field is mainly toroidal). The configuration represents a near-minimum energy state to which a plasma relaxes under proper constraints [7]. The toroidal field changes sign in the outer part of the plasma, a feature which gives the name to the configuration. Such field reversal, which improves the MHD stability of the configuration, is spontaneously generated by the plasma, and is maintained against resistive diffusion by the dynamo process [8]. This is achieved through the action and nonlinear coupling of several resistive MHD modes, which give rise to a high level of magnetic turbulence (of the order of 1% of the average field in present day experiments, i.e., two orders of magnitude larger than in tokamaks). This high fluctuation level makes the RFP very suited for the study of MHD turbulence properties, mainly for their magnetic part. The magnetic turbulence has been demonstrated to be the main cause of energy and particle transport in the RFP core, whereas at the edge its contribution is still under investigation. In this region the electrostatic turbulence has been proved to give an important contribution to the particle transport [9]. It is worth mentioning that a recent investigation of the edge electrostatic turbulence in different fusion devices (including RFP and tokamaks) has shown the existence of long range time and space correlations [10].

The measurements described in this paper have been obtained in the (RFX) experiment, which is the largest RFP presently in operation (major radius 2 m, minor radius 0.457 m) [11]. RFX is designed to reach a plasma current of 2 MA, and currents up to 1 MA have been obtained up to now. The measurements were performed in low currents discharges (300 kA) using a magnetic probe inserted in the edge plasma. The probe consists of a coil housed in a boron nitride protecting head. The coil measures the time derivative $\partial_t B$ of the radial component $B(t)$ of the magnetic field. The radial direction in this case goes from the core plasma to the edge. The sampling frequency of the measurements is 2 MHz. Measurements have been collected at different values of the normalized radius $r/a$ ($r/a = 1$ identifies the location of the last magnetic flux surface). In RFX two different components of the magnetic fluctuations can be identified: a localized and stationary magnetic perturbation, originated by the tearing modes responsible for the dynamo, which tend to be phase-locked and locked to the wall [12], and a high
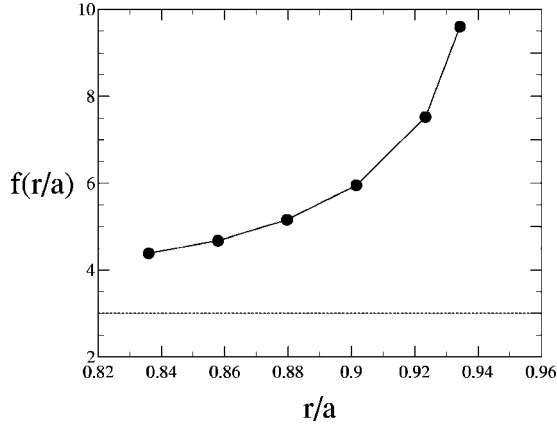
FIG. 1. We show the values of the flatness $f(r/a)$ for the derivative of the radial magnetic field, as a function of the insertion $r/a$. The Gaussian value ($f=3$) is shown as a dotted line.

frequency broadband activity, which is investigated here. All measurements presented were made away from the stationary perturbation.

We start by looking at the statistical properties of the normalized variables $s(t) = \partial_t B / \sqrt{\langle (\partial_t B)^2 \rangle}$ (brackets being time averages). In Fig. 1 we show the flatness of these stochastic variables $f = \langle s^4 \rangle / [\langle s^2 \rangle]^2$ for some positions $r/a$. As can be seen $f(r/a)$ is higher than the Gaussian value and tends to decrease as $r/a$ increases. This is a first rough evidence that the observed magnetic field is intermittent; that is, the time evolution of $\partial_t B$ is dominated by strong magnetic fluctuations. The intermittency (say the departure from a Gaussian statistics) is more visible near the external wall.

To get some insight into the nature of intermittency actually present in the fusion device, following the usual analysis currently made in fluid flows [2], we investigate the scaling behavior of the stochastic variables $\delta B(\tau) = B(t+\tau) - B(t)$, which represents characteristic fluctuations across turbulent structures at the scale $\tau$. For each position within the device, we can study the statistical behavior of fluctuations at different scales $\tau$. The interest of this resides in the fact that, if we introduce a scaling law for magnetic fluctuations $\delta B(\tau) \sim \tau^h$ as MHD equations seem to indicate [13], a scale variation $\tau \to \lambda \tau$ ($\lambda$ is the parameter which defines the change of scale) leads to

$$\delta B(\lambda \tau) = \lambda^{-h} \delta B(\tau).$$

This is interpreted as an ''equality in law'' [2]; that is, the right-hand side of the equation has the same statistical properties as the left-hand side. If $h$ is constant we can easily show that the PDFs of the normalized stochastic variables $\delta b_\tau = \delta B(\tau) / \sqrt{\langle [\delta B(\tau)]^2 \rangle}$ collapse to a unique PDF, independent on the scale $\tau$. This is true in a pure self-similar (fractal) case. On the contrary, we must invoke the multifractal model to describe intermittency [2], which is introduced by defining a range of values of $h$.

In Fig. 2 we report the PDFs of $\delta b_\tau$ at different scales for a given value of $r/a$. As can be seen the PDFs do not collapse to a single curve, but follow a characteristic scaling behavior that is visible for all values of $r/a$. At large scales the PDF are almost Gaussian, and the wings of the distribu-
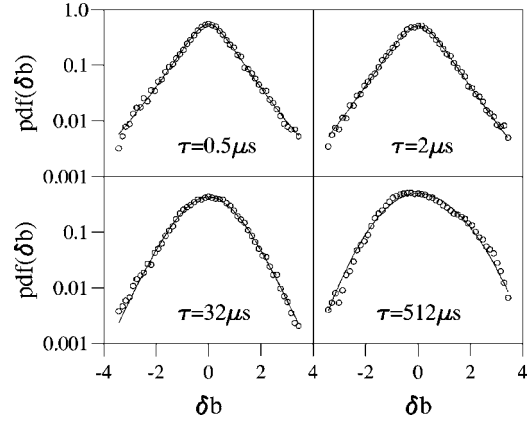


FIG. 2. We show the PDFs of the normalized magnetic fluctuations for four different scales, at a given position $r/a = 0.95$. The solid line represents the fit made with the convolution function.

tions grow up as the scale becomes smaller. Stronger events at small scales have a probability of occurrence greater than that they would have if they were distributed according to a Gaussian function. This behavior, that is, the presence of self-affine fields, is at the heart of the phenomenon of intermittency as currently observed in fluid flows [2,14].

The behavior of PDFs against the scale can be described by introducing a given shape for the distribution. At each scale $\tau$ the PDF of $\delta b_\tau$ can be represented as a convolution of Gaussian functions of widths $\sigma$ whose distribution is given by a function $G_\lambda(\sigma)$

$$P(\delta b_\tau) = \frac{1}{\sqrt{2\pi}} \int_0^\infty G_\lambda(\sigma) \exp(-\delta b_\tau^2 / 2\sigma^2) \frac{d\sigma}{\sigma}, \quad (1)$$

which can be interpreted in the framework of a cascade model as the signature of an underlying multiplicative process [14–16]. We use a log-normal ansatz

$$G_\lambda(\sigma) = \frac{1}{\sqrt{2\pi}\lambda} \exp\left(-\frac{\ln^2 \sigma/\sigma_0}{2\lambda^2}\right), \quad (2)$$

even if other functions do not give very different results [14]. The free parameter $\lambda^2$ represents the width of the distribution $G_\lambda$, while $\sigma_0$ is the most probable value of $\sigma$. The scaling behavior of $P(\delta b_\tau)$ is translated in the scaling varia-
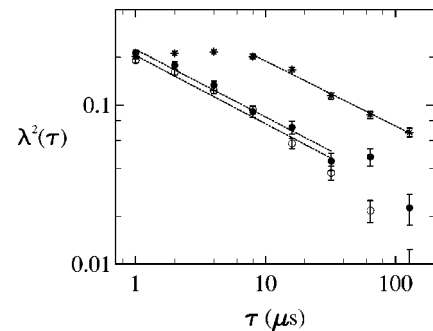


FIG. 3. Scaling behavior of the exponent $\lambda^2(\tau, r/a)$ for three different insertions $r/a$, namely: $r/a = 0.97$ (closed circles), $r/a = 0.95$ (open circles), and $r/a = 0.91$ (stars).
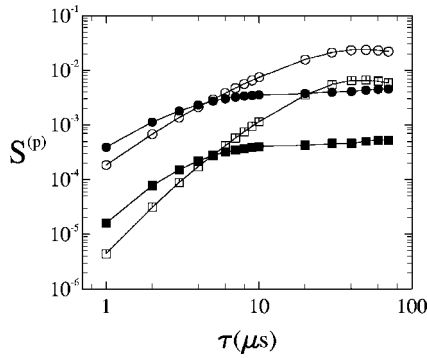
FIG. 4. Structure functions $S_\tau^{(p)}$ are shown for $p=2$ (circles) and $p=3$ (squares). Open symbols refer to the position $r/a=0.96$, closed symbols refer to $r/a=0.86$.



FIG. 5. Normalized scaling exponents $\zeta_p$ of the structure functions are shown as a function of $p$, for different insertion points $r/a$. Error bars, about 5% of the exponent values, are not displayed for clarity. The K41 scaling $\zeta_p \sim p/3$ is also reported for comparison.

tion of the parameter $\lambda^2$ [14,16]. In fact, when the PDF is Gaussian $\lambda^2=0$ ($G_\lambda$ becomes a $\delta$ function centered around $\sigma_0$), while the departure from a Gaussian function increases as $\lambda^2$ increases. In Fig. 2 we report as solid line a fit of the data with Eq. (1). A satisfactory agreement at all scales is evident.

Looking at the scaling laws for $\lambda^2$, at different insertion points $r/a$, it can be seen (Fig. 3) that $\lambda^2$ displays a power law behavior

$$\lambda^2(\tau, r/a) = \Lambda^2(r/a)\tau^\beta$$

all over the observed time scales, for insertion points near the external wall. On the contrary, measurements more inside the device show a saturation of intermittency at scales $\tau_S \simeq 10$ $\mu$s. The values of $\beta$ we find are of the order of $\beta \simeq 0.42 \pm 0.03$, close to that found for the velocity field both in fluid flows and in the solar wind turbulence [14,16], but higher than the value found for the magnetic field intensity in the solar wind [16]. Finally the absolute values for $\lambda^2$ are decreasing going from the wall inside the device. Namely, we found $\lambda_{max}^2 = 0.21 \pm 0.01$ for $r/a = 0.98$, and $\lambda_{max}^2 = 0.086 \pm 0.006$ for $r/a = 0.84$. This is a further confirmation of the stronger intermittency near the external wall. All the error bars have been estimated starting from a Poisson statistical uncertainty on the PDF's value.

A complementary analysis of intermittency can be performed by calculating the scaling exponents of the structure functions, say of the $p$th moments of fluctuations $S_\tau^{(p)} = \langle \delta b_\tau^p \rangle$ (brackets are defined as time averages). In Fig. 4 we report the structure functions $S_\tau^{(p)}$, for two values of $p$, and for two different position $r/a$. The differences for different positions are evident, and represent a signature of the absence of universality.

To calculate the scaling exponents, we use the generalized scaling introduced by Benzi et al. [17], which has been found to be useful also in magnetohydrodynamic turbulence [13,18], thus obtaining the normalized scaling exponents $\zeta_p$ defined through $S_\tau^{(p)} \sim [S_\tau^{(3)}]^{\zeta_p}$.

In Fig. 5 we report the scaling exponents obtained for some insertions $r/a$. The behavior of $\zeta_p$ against $p$ shows that scaling exponents are anomalous, say, they are different from
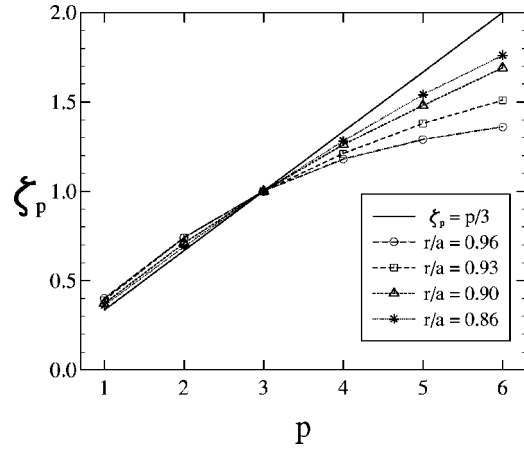
the usual $p/3$ Kolmogorov's law. Note once more that the strength of intermittency, measured through the difference between $\zeta_p$ and $p/3$, is greater near the wall. In conclusion, scaling laws for PDFs of magnetic fluctuations, and anomalous scalings for structure functions, are found everywhere in the outer plasma region of the RFX thermonuclear fusion experiment. We find that the anomaly of scaling exponents, as well as scaling laws for PDFs, strongly depend on the position inside the plasma, so that magnetic turbulence inside the device is not universal, as far as scaling laws are concerned. Possible reasons for this are the presence of the first wall, the presence of the toroidal field reversal (which takes place at $r/a \simeq 0.9$), or the strongly sheared plasma flow measured in the RFX edge [19]. Concerning this latter option, it is worth mentioning that in principle different plasma velocities in different points would only affect the relationship between time and spatial scales obtained through Taylor hypothesis, and not the PDF scaling properties. However, the eddy breaking effect induced by a velocity shear is well known to affect electrostatic turbulence in fusion plasmas [20,9], and an influence on MHD turbulence can also be envisaged, either directly or through nonlinear coupling to electrostatic modes. If this is not the case, the reason for the observed differences could be perhaps found in the conjecture of Farge [21]. She proposed that turbulence could be described by interwoven sets of both intermittent structures and background Gaussian flow on each characteristic scale. The nature of the intermittent structures can evidently be influenced by walls [3], and/or current sheets associated with field reversal [22]. We are actually reviewing and testing this idea on the RFX device in order to identify structures which generate intermittency. Since a reduction of magnetic fluctuations has been linked to improvements in the energy confinement [23], a better understanding of the generation of intermittency through structures could improve the understanding confinement in physics.

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